1) If $g(x) = \frac{1}{32}x^4 - 5x^2$, find $g'(4)$.

a) -72 b) -32 c) -24 d) 24 e) 32

$\frac{1}{8}x^3 - 10x$

$\frac{1}{8}(4)^3 - 10(4)$

$8 - 40 = -32$

2) Evaluate $\lim_{h \to 0} \frac{5(\frac{1}{2}+h)^4 - 5(\frac{1}{2})^4}{h}$.

$f(x) = 5x^4$

$f'(\frac{1}{2}) = 20(\frac{1}{2})^3 \rightarrow 20(\frac{1}{8}) = \frac{20}{8} = \frac{5}{2}$

a) $\frac{5}{2}$ b) $\frac{5}{16}$ c) 40 d) 160 e) Limit does not exist

$\underline{108}$ $4(3)^3 - 7(3)^2$

-63

45

3) An equation of the line tangent to $y = 4x^3 - 7x^2$ at $x = 3$ is

$(3, 45)$

$y' = 12x^2 - 14x \rightarrow 12(3)^2 - 14(3)$

$108 - 42$

$\underline{66}$ slope

a) $y + 45 = 66(x+3)$ b) $y = 66(x-3)$

c) $y - 45 = 66(x-3)$ d) $y - 45 = \cancel{-\frac{1}{66}}(x-3)$

e) $y = 66x$

Weekly Review 6_0910

1) Let f be a continuous, differentiable, and monotonic function on the domain [3, 8]. The table shows four function values of f.

x	3	4	6	7
$f(x)$	-4	1	5	8

3pts

Which of the following statements must be true? Explain your choice.

I. $f(8) > 9$ No, could plateau after $x=7$

II. $f'(5) > 0$ No, plateau, yes ≥ 0

True → III. $f'(c) = 3$ for exactly one c in [3, 7]

mean value thrm

$f'(c)=3$ [3, 7] → secant line $m = \frac{8-4}{7-3} = \frac{12}{4} = 3$

(A) II only (B) II and III only (C) III only

(D) I and III only (E) I, II, III (F) None of these

2) Let g be a function defined and continuous on the closed interval $[a, b]$. If g has a local minimum at c where $a < c < b$, which of the following statements must be true? Explain your choice.

3pts.

I. If $g'(c)$ exists, then $g'(c) = 0$ yes, def. min deriv.=0 if exists

II. $g(c) < g(b)$ No

III. g is monotonic on $[a, b]$ – No, no extrema → sign change

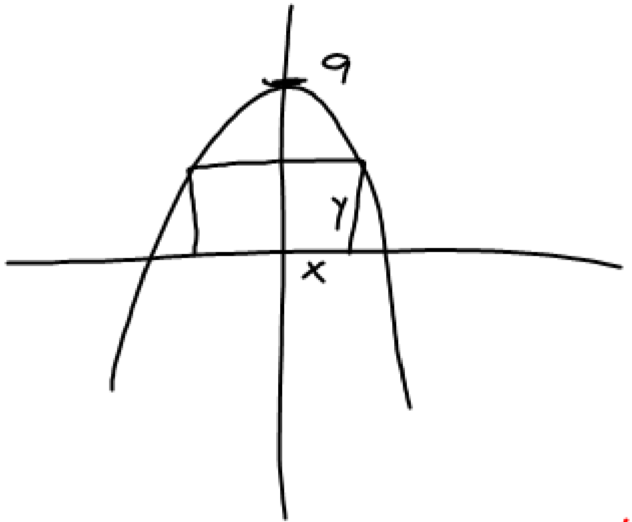
(A) I only (B) II only (C) III only

(D) I and II only (E) I and III only (F) None of these

3) A rectangle is inscribed under the graph of $h(x) = 9 - x^2$ and above the x-axis. Find the maximum possible area for that rectangle.

4) An open-topped box with a square base must be constructed with a volume of 12 cubic inches. What dimensions use the least amount of material? (*Hint:* Define your variables for dimensions of the box. Now write two equations, one for volume and one for surface area. Which one will be optimized?)

5) From an 8 inch by 10 inch rectangular sheet of paper, square of equal size will be cut from each corner. The flaps will then be folded up to form an open-topped box. Find the maximum possible volume of the box.



$A = 2xy$

$A = 2x(9 - x^2)$

$A = -2x^3 + 18x$

work 1 pt

$A' = -6x^2 + 18$

$-6x^2 + 18 = 0$

$x^2 = 3$

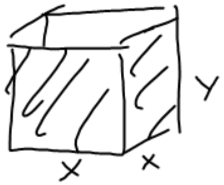
$x = \pm\sqrt{3}$

$x = \sqrt{3}$ 1pt

$y = 9 - (\sqrt{3})^2$

$y = 6$

$A = 12\sqrt{3}$ 1pt



$V = 12 = x^2 \cdot y$

$y = \frac{12}{x^2}$

$SA = x^2 + 4xy$

$SA = x^2 + 4\cancel{x}\left(\frac{12}{x^{\cancel{2}}}\right)$

$SA = x^2 + \frac{48}{x}$

$SA = \frac{x^3 + 48}{x}$

work
1pt

$SA' = \frac{x(3x^2) - (x^3+48)\cdot 1}{x^2}$

$0 = \frac{3x^3 - x^3 - 48}{x^2}$

$2x^3 - 48 = 0$

$x^3 = 24$

$x = \sqrt[3]{24}$

$\boxed{x = 2\sqrt[3]{3}}$

1pt

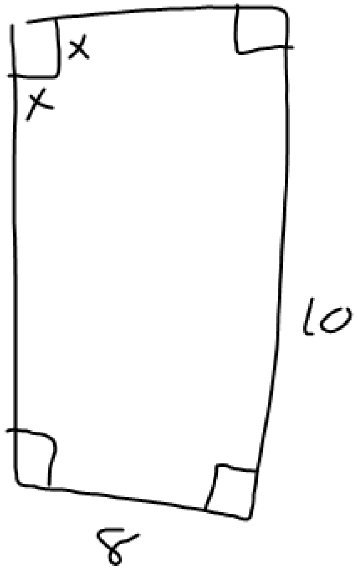
$y = \frac{12}{x^2}$

$y = \frac{12}{(2\sqrt[3]{3})^2} = \frac{12}{4\sqrt[3]{3^2}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \frac{12 \cdot \sqrt[3]{3^4}}{4 \cdot 3^2}$

$\cancel{\frac{12 \cdot 3\sqrt[3]{3}}{4 \cdot 3^2}} = \sqrt[3]{3}$

y

1pt



$V = x(8-2x)(10-2x)$

D: $(0, 4)$

$V = x(4x^2 - 36x + 80)$

$V = 4x^3 - 36x^2 + 80x$

$V' = 12x^2 - 72x + 80 = 0$

$= 4(3x^2 - 18x + 20)$

↳ Quad. formula

$$\frac{18 \pm \sqrt{(-18)^2 - 4(3)(20)}}{2(3)}$$

≈ 4.53 Not in Domain

$\boxed{1.47} = x$

$V(1.47) = 52.51 \text{ in}^3$

1pt x
1pt V
1pt work

$\frac{9}{15} = 60\%$

Sect. 1.2 #55, 56

optimize analytically w/ calculus

$\frac{1}{3}\pi r^2 h$

$$\boxed{r = \frac{8\pi - x}{2\pi}}$$

$$\boxed{h = \sqrt{16 - \left(\frac{8\pi - x}{2\pi}\right)^2}}$$

$Y_1 = \frac{(8\pi - x)}{(2\pi)}$

$V = \frac{1}{3}\pi r^2 h$

$Y_2 = \text{nderiv}\,(Y_1)$

$$\frac{dv}{dx} = \frac{\pi}{3}\left[r^2 \frac{dh}{dx} + h \cdot \frac{dr}{dx}\right]$$

$Y_3 = \text{my}\ \frac{dr}{dx}$